Exercise 58

For the following exercises, sketch a graph of the quadratic function and give the vertex, axis of symmetry, and intercepts.

$$f(x) = 4x^2 - 12x - 3$$

Solution

In order to more easily graph the quadratic function, write it in vertex form by completing the square. The following algebraic identity is necessary.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Factor the coefficient of x^2.

$$f(x) = 4\left(x^2 - 3x - \frac{3}{4}\right)$$

Notice that $2B = -3$, which means $B = -\frac{3}{2}$ and $B^2 = \frac{9}{4}$. Add and subtract $\frac{9}{4}$ within the parentheses and use the identity.

$$\begin{aligned} f(x) &= 4\left[\left(x^2 - 3x + \frac{9}{4}\right) - \frac{3}{4} - \frac{9}{4}\right] \\ &= 4\left[\left(x + \left(-\frac{3}{2}\right)\right)^2 - 3\right] \\ &= 4\left(x - \frac{3}{2}\right)^2 - 12 \end{aligned}$$

Therefore, the vertex is $\left(\frac{3}{2}, -12\right)$, and the axis of symmetry is $x = \frac{3}{2}$. To determine the y-intercept, set $x = 0$.

$$f(0) = 4\left(0 - \frac{3}{2}\right)^2 - 12 = 4\left(\frac{9}{4}\right) - 12 = -3$$

Therefore, the y-intercept is $(0, -3)$. To get the x-intercept, set $y = 0$ and solve the equation for x.

$$0 = 4\left(x - \frac{3}{2}\right)^2 - 12$$

$$12 = 4\left(x - \frac{3}{2}\right)^2$$

$$3 = \left(x - \frac{3}{2}\right)^2$$

Take the square root of both sides.

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{3}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - \frac{3}{2}$.

$$\left|x - \frac{3}{2}\right| = \sqrt{3}$$

Remove the absolute value sign by placing ± on the opposite side.

$$x - \frac{3}{2} = \pm\sqrt{3}$$

Add 3/2 to both sides.

$$x = \frac{3}{2} \pm \sqrt{3}$$

This means $x = \left\{\frac{3}{2} - \sqrt{3}, \frac{3}{2} + \sqrt{3}\right\}$, and the x-intercepts are $\left(\frac{3}{2} - \sqrt{3}, 0\right)$ and $\left(\frac{3}{2} + \sqrt{3}, 0\right)$. A graph of the function is shown below.